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# Ramapuram campus

# Department of Mathematics

18MAB302T- DISCRETE MATHEMATICS

Year/Sem: III/V Branch: CSE,ECE,EEE

**UNIT-4 -GROUP THRORY**

1. Let be the group then for each the value of is
2. (b) (c) (d) **Ans: d**

**Solution:** (By Associative Law)

= (By Inverse Law)

= (By Identity Law)

=

Hence Inverse of is

1. Let then under usual multiplication is
2. Group (b) Sub Group (c) Cyclic Group (d) Not a Group **Ans: a**

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| --- | --- | --- |
| **.** | 1 | -1 |
| 1 | 1 | -1 |
| -1 | -1 | 1 |

**Solution:** Cayley Table of is

From the above table satisfies Closure law, since multiplication is associative in any number set, it is true here also. Hence associative axiom is satisfied. 1 is the Identity element. Inverse of 1 is 1 and Inverse of -1 is -1. Hence is a group.

1. Let be the set of all non-zero real numbers defined by the binary operator

, & is Abelian Group. Then Identity element of is

1. 4 (b) 2 (c) 1 (d) 0 **Ans: b**

**Solution:**

1. The fourth root of unity where forms an Abelian group under multiplication. Then Inverse of are
2. (b) 1,1 (c) (d) **Ans: d**

**Solution:** Cayley Table

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From the above table 1 is the Identity element.

Inverse of are

1. Which of the following statements are true?
2. Identity element of a group is unique and Inverse of each element is finite.
3. Identity element of a group is unique and Inverse of each element is same.
4. Identity element of a group is unique and Inverse of each element is unique.
5. Identity element and Inverse element are equal for any group.
6. I,II (b) III (c) IV (d) III&IV **Ans: b**

**Solution:** From Properties of Group.

1. An algebraic structure \_\_\_\_\_\_\_\_\_ is called a semigroup.  
   a) b) c) d) **Ans: a**

**Solution:** An algebraic structure is called a semigroup if for all or the elements follow associative property under .

1. A cyclic group is always \_\_\_\_\_\_\_\_\_  
   a) abelian group b) monoid c) semigroup d) subgroup **Ans: a**  
   **Solution:** A cyclic group is always an abelian group but every abelian group is not a cyclic group. For instance, the rational numbers under addition is an abelian group but is not a cyclic one.
2. If then is \_\_\_\_\_\_\_\_\_\_\_\_\_

a) Monoid b) Abelian group c) Commutative semigroup d) Cyclic group

**Ans: c**  
  
**Solution:** Let x and y belongs to a group .Here closure and associativity axiom holds simultaneously. Let be an element in G such that

So, identity axiom does not exist but commutative property holds. Thus, is a commutative semigroup.

1. From the group , order of the element 4 is
2. 0 (b) 1 (c) 3 (d) 5 **Ans: d**

**Solution:** Identity element of is 0

1. From the Multiplicative group the order of is
2. 1 (b) 2 (c) 3 (d) 0 **Ans: c**

**Solution:** Identity element of is 1

Hence

1. If is a cyclic group of order 73, then number of generator of G is equal to \_\_\_\_\_\_

a) 89 b) 23 c) 72 d) 17 **Ans: c**

**Solution:** We need to find the number of co-primes of 73 which are less than 73. As 73 itself is a prime, all the numbers less than that are co-prime to it and it makes a group of order 72 then it can be of {1, 3, 5, 7, 11….}.

1. The dihedral group having order 6 can have degree \_\_\_\_\_\_\_\_\_\_\_\_\_

a) 3 b) 26 c) 326 d) 208 **Ans: a**

**Solution:** A symmetric group on a set of three elements is said to be the group of all permutations of a three-element set. It is a dihedral group of order six having degree three.

1. Suppose (2, 5, 8, 4) and (3, 6) are the two permutation groups that form cycles. What type of permutation is this?  
   a) odd b) even c) acyclic d) prime **Ans: b**

**Solution:** There are four permutations (2, 5), (2, 8), (2, 4) and (3, 6) and so it is an even permutation.

1. Invariant permutations of two functions can form \_\_\_\_\_\_\_\_\_\_  
   a) groups b) lattices c) graphs d) rings **Ans: a**

**Solution:** Suppose, there are two functions f1 and f2 which belong to the same equivalence class since there exists an invariant permutation say, (a permutation that does not change the object itself, but only its representation), such that: . So, invariant permutations can form a group, as the product (composition) of invariant permutations is again an invariant permutation.

1. The transpositions of the permutation are
2. (1 6) (2 5) (2 3) (b) (1 6) (1 5) (1 3)

(c) (2 5) (2 3) (2 6) (d) (1 3) (1 5) (1 6) **Ans: a**

**Solution:** = (1 6) (2 5 3) (4) (7)

= (1 6) (2 5 3)

= (1 6) (2 5) (2 3)

1. Non-trivial subgroups of are
3. **Ans: b**

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**Solution:**

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Both are closed under Hence they are subgroups.

1. Let G be a finite group with two sub groups such that and . Determine the value of   
   a) 1 b) 56 c) 14 d) 78

**Solution:** We know that gcd(56, 123)=1. So, the value of |M⋂N|=1

1. Let K be a group with 8 elements. Let H be a subgroup of K and H<K. It is known that the size of H is at least 3. The size of H is \_\_\_\_\_\_\_\_\_\_  
   a) 8 b) 2 c) 3 d) 4 **Ans: d**  
     
     
   **Solution:** For any finite group G, the order (number of elements) of every subgroup L of G divides the order of G. G has 8 elements. Factors of 8 are 1, 2, 4 and 8. Since given the size of L is at least 3(1 and 2 eliminated) and not equal to G(8 eliminated), the only size left is 4. Size of L is 4.
2. A function is defined by and is called \_\_\_\_\_\_\_\_\_\_\_\_\_  
   a) isomorphic b) homomorphic c) cyclic group d) heteromorphic

**Ans: a**

Let and are two groups. The mapping f is said to be isomorphism if two conditions are satisfied 1) is one-to-one function and onto function and 2) satisfies homomorphism.

1. How many different non-isomorphic Abelian groups of order 8 are there?  
   a) 5 b) 4 c) 2 d) 3 **Ans: c**

**Solution:** The number of Abelian groups of order  (let, is prime) is the number of partitions of . Here order is 8 i.e. 23 and so partition of 3 are {1, 1} and {3, 0}. So number of different abelian groups are 2.

1. Let be the set of Integers with Binary operators defined by ,

. Then is

Commutative Ring with Identity

Non Commutative Ring

Commutative Ring without Identity

Not a Ring **Ans: a**

**Solution:**  
 is Abelian Group,  
 Satisfies the properties of Ring along with Identity and commutative.

1. If is a Field with respect to addition and Multiplication. Then the Inverse of each element of with respect to Addition is
2. (b) (c) (d) **Ans:d**

**Solution:** Since is an abelian group ,

1. Let and be two Rings. given by is
2. Ring Homomorphism
3. Group Homomorphism
4. Not a Ring Homomorphism
5. Group Isomorphism **Ans: c**

**Solution:**

1. The only Idempotent elements of an Integral Domain are
2. 0 &1 (b) 0&2 (c) 1&2 (d) 1&3 **Ans: a**

**Solution:** Let be an Integral domain. Let be an Idempotent element

Then

Since has no Zero Divisors only.

1. If and then is
2. 6 (b) 4 (c) 3 (d) 5 **Ans: b**

**Solution:**

1. If the message and let then is
2. (0 0 0 0 0) (b) (1 0 1 1 0) (c) ( 0 1 0 1 1) (d) (1 1 1 0 1) **Ans: d**

**Solution :**

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